2020 5th International Workshop on Materials Engineering and Computer Sciences (IWMECS 2020)

Linearization Model and State Space Model Analysis of Inverted Pendulum System

Xiao Cong[1,*], Liming Gong [2], Junwei Lei [3], and Yuliang Chen [3]

[1]Shandong Businiess Institute, Yantai, 264001, China

[2]News and culture room of teaching and research guarantee center, Naval Aviation University, Yantai, 264001, China

[3]College of Coastal Defense, Naval Aviation University, Yantai, 264001, China

email: cx-1217@163.com

Keywords: Pendulum System; Simulation; Linearization; State Space; Model

Abstract: In this paper, aiming at the nonlinear inverted pendulum system model, the linearization model is analyzed, and the small perturbation method is mainly used to linearize the inverted pendulum system model. On this basis, the state space model is analyzed, and the state space equation is established by choosing appropriate physical variables as state variables. Finally, the matrix state space equation description form is obtained, which provides a good model basis for the subsequent digital simulation analysis and step response analysis of the system.

1. Introduction

As a typical non-linear, multi-variable, strong coupling and high order unstable system, inverted pendulum system requires high control accuracy and real-time performance for its stability control. Therefore, inverted pendulum has been an ideal experimental platform for the study of control theory and control engineering since its emergence. In this paper, the linear inverted pendulum is taken as the research object [1-5]. Firstly, the background and significance of the inverted pendulum system research, the classification of inverted pendulum and the current situation of its development at home and abroad are introduced in combination with sliding mode variable structure control. Then, the mathematical model of the linear inverted pendulum is deduced.

The stability of inverted pendulum system is similar to that of space vehicle control and various servo platforms. It is also an abstraction of the control problem of the center of gravity at the top and the fulcrum at the bottom in daily life [6-8]. Through the study of the inverted pendulum system, it can also reveal many problems in industrial applications and robotic systems, such as the non-linear characteristics of the system, the saturation phenomenon of the actuator, the calibration error phenomenon of the sensor, and the constraints of some physical variables in the system.

Therefore, the control problem of inverted pendulum system is often used to test the effectiveness of new control theory and method, and it is an ideal experimental method in control theory [9-11].

Therefore, the research of variable structure control method based on inverted pendulum mechanism has important theoretical significance and practical application value. The research of this subject is based on this background.

2. Linearization Processing and Transfer Function of Single Inverted Pendulum System Model

Assuming that $\theta = \pi + \phi$, assume ϕ is very small compared with 1 (the unit is radian), that is, far less than 1, approximation can be made.

$$\cos\theta = -1, \sin\theta = -\phi, \left(\frac{d\theta}{dt}\right)^2 = 0 \tag{1}$$

Let u represent the input force F of the controlled object, and the system model of pendulum is linearized as

$$\begin{cases}(I+ml^2)\ddot{\phi}-mgl\phi=ml\ddot{x}\\(M+m)\ddot{x}+b\dot{x}-ml\ddot{\phi}=u\end{cases}\tag{2}$$

Where $I=\frac{1}{3}ml^2$.

Therefore, the equation of state of the inverted pendulum is:

$$\begin{cases}\ddot{\theta}=\dfrac{3g(M-m)}{l(4M+m)}\theta+\dfrac{-3}{l(4M+m)}F\\ \ddot{x}=\dfrac{-3mg}{4M+m}\theta-\dfrac{4}{(4M+m)}F\end{cases}\tag{3}$$

3. Derivation of Transfer Function of Single Inverted Pendulum

By using Laplace transformation of equation (3), the following results are obtained:

$$\begin{cases}(I+ml^2)\Phi(s)s^2-mgl\Phi(s)=mlX(s)s^2\\(M+m)X(s)s+bX(s)s-ml\Phi(s)s^2=U(s)\end{cases}\tag{4}$$

When the initial condition is 0, the first equation of the system can be obtained by solving the output angle.

$$X(s)=\left[\frac{(I+ml^2)}{ml}-\frac{g}{s^2}\right]\Phi(s)\tag{5}$$

By substituting the above formula into the second equation in (4), we obtain that:

$$(M+m)\left[\frac{(I+ml^2)}{ml}-\frac{g}{s}\right]\Phi(s)s^2+b\left[\frac{(I+ml^2)}{ml}+\frac{g}{s^2}\right]\Phi(s)s-ml\Phi(s)s^2=U(s)\tag{6}$$

After sorting out, we can get:

$$\frac{\Phi(s)}{U(s)}=\frac{\frac{ml}{q}s}{s^3+\frac{b(I+ml^2)}{q}s^2-\frac{(M+m)mgl}{q}s-\frac{bmgl}{q}}=\frac{num}{den}\tag{7}$$

Where $q=[(M+m)(I+ml^2)-(ml^2)]$.

4. Derivation of State Space Equation

The state equation of common linear system can be written as:

$$\begin{cases}\dot{X}=AX+Bu\\y=CX+Du\end{cases}\tag{8}$$

Where A is the state matrix. B is the input matrix.C is the output matrix.D is a forward feed matrix.

The equations (3) are solved as

$$
\begin{cases}
\dot{x} = \dot{x} \\
\ddot{x} = \dfrac{-(I+ml^2)b}{I(M+m)+Mml^2}\dot{x} + \dfrac{mgl(m+M)}{I(m+M)+Mml^2}\phi + \dfrac{(I+ml^2)}{I(M+m)Mml^2}u \\
\dot{\phi} = \dot{\phi} \\
\ddot{\phi} = \dfrac{-(I+ml^2)b}{I(M+m)+Mml^2}\dot{x} + \dfrac{mgl(m+M)}{I(m+M)+Mml^2}\phi + \dfrac{(I+ml^2)}{I(M+m)Mml^2}u
\end{cases}
\tag{9}
$$

After sorting out, the state space equation of the system is as follows:

$$
y = \begin{bmatrix} x \\ \phi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \tag{10}
$$

According to the mathematical model of linear inverted pendulum, the controlled object is a single input (force F) and double output system (displacement of car, angle of pendulum rod).

Set $u' = \ddot{x}$, then

$$
\begin{aligned}
\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{3g}{4l} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{3}{4l} \end{bmatrix} u' \\
y = \begin{bmatrix} x \\ \phi \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u'
\end{aligned}
\tag{11}
$$

Finally, we get the whole state space equation model of pendulum system and transfer function based on above linear model.

5. Conclusion

On the basis of the non-linear first-order inverted pendulum model, the process of linearizing the linear system described by its linear differential equation with small perturbations is given in this paper. The main use is to use linear function to approximate the non-linear function when the pendulum angle is small or large, or when the balance angle changes slightly. The simplified linearized model and transfer function are obtained. Finally, the state space matrix of the simplified linearized model is obtained, which provides a good model basis for the control and Simulation of the inverted pendulum.

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